Exponents and logarithms

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November 2021

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1 Exponent and logarithm rules

1. Exponent form and logarithm form: $a^b = c \iff \log_a c = b$
   - A logarithm is the exponent, b, of the expression written in exponential form. You should build an intuition for logarithm rules by focusing on what happens to the exponents in corresponding exponent rules.
   - $\log_b 1 = 0$ because $b^0 = 1$.
   - $\log_b b = 1$ because $b^1 = b$.
   - $\log_b b^k =$_ because ______. [1]
   - $b^{\log_b k} = k$ because b raised to the power b needs to be raised to to equal k is k.
   - The domain of $\log x$ is $(0, \infty)$. The range is $(-\infty, \infty)$ Why [2]? Hint: convert the problem to exponential form.
2. Default bases: $\log = \log_{10}$ and $\ln = \log_e$. In computer science, $\log = \log_2$.
3. Multiplication equivalent to addition:
   - $a^b \cdot a^c = a^{b+c}$
   - $\log(ab) = \log a + \log b$

[1] k, converting the log expression to exponent form $b^? = b^k$ shows that k goes in the blank

[2] Assume an arbitrary base of 10. In exponential form, we have the problem $10^? = x$. From this equation, it should be clear that $x \leq 0$ is not possible, so the domain is $(0, \infty)$. The range is any number that can take the place of the blank, which is any real number.

4. Division equivalent to subtraction:
   - $\frac{a^b}{a^c} = a^{b-c}$
   - $\log \frac{a}{b} = \log a - \log b$
5. Exponentiation:
   - $(a^b)^c = a^{bc}$
   - $\log(b^c) = c \log b$
6. Change of base formula:
   - $\log_b a = \frac{\log_c a}{\log_c b}$, where c is arbitrary.
   - In English, you can calculate a logarithm with any base by typing $\log a / \log b$ on your calculator, where $\log = \log_{10}$.
   - Proof

2 Example problems

1. Simplify $x^3 = e^{57}$.
2. If $x = (\frac{a^2 b^3}{a+c^2})^5$, what is $\log x$? Simplify.

3 Solutions

1. $(x^3)^{\frac{1}{3}} = (e^{57})^{\frac{1}{3}} \implies x = e^{19} \implies \ln x = 19$
2. $x = (\frac{a^2 b^3}{a+c^2})^5 = \frac{a^{10} b^{15}}{(a+c^2)^5} \implies \log x = \log \frac{a^{10} b^{15}}{(a+c^2)^5} = 10 \log a + 15 \log b - 5 \log (a + c^2)$